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VORTEX INTERPRETATION OF THE THEORY OF FUNCTIONS OF A COMPLEX VARIABLE

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The following is a digest of a work which represents novel mathematical representations of interest to hydromechanics and erodynamics. The author's proofs are eliminated here.

The theory of functions of a complex variable is a powerful tool in the study of two-dimensional potential flows of an ideal incompressible fluid. It is completely natural to raise the question: Does there exist a clars of threedimensional flows which can be studied with the aid of the theory of functions of a complex variable?

It is easily shown that the potential motion of a holonomic system in a configurational space is the projection of a vortex from a space with the number of dimensions one greater than the natural space of the system (see Arzhanykh, Iz Ak Nauk Uzbek SSR, No 3, 1949).

In applications to hydrodynamics, this theorem permits one to construct a wide class of vortex flows in three-dimensional space which are defined by a function of a complex variable. Moreover, the theory of functions of a complex variable acquires a vortex interpretation.

Theorem 1

The class of three-dimensional vortex flows of an ideal incompressible homogeneous fluid under the action of potential forces corresponds to an analytic function $F(\zeta,t)=\varphi(x,y;t)+i\psi(x,y;t)$ of a complex variable $\zeta=x+iy$, which depends on time t as a parameter. If f(x,y;t)=a and g(x,y;t)=b, namely the integrals of the system of differential equations $\dot{x}=\partial\phi/\partial x$ and $\dot{y}=\partial\phi/\partial y$, then the velocity field of the corresponding class of flows will be:

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$$V_{x} = \partial \varphi / \partial x$$
, $V_{y} = \partial \varphi / \partial y$, $V = \chi(t) + h(f,g)$ (1)

and the vortex field will be:

$$\Omega_{x} = \frac{\partial h}{\partial f} \cdot \frac{\partial f}{\partial y} + \frac{\partial h}{\partial g} \cdot \frac{\partial g}{\partial y}, \quad \Omega_{y} = -\frac{\partial h}{\partial f} \cdot \frac{\partial f}{\partial x} - \frac{\partial h}{\partial g} \cdot \frac{\partial g}{\partial x}, \quad \Omega = 0,$$
(2)

Pressure is defined by the formula:

$$\rho = c(t) - \rho \frac{\partial \varphi}{\partial t} + \rho \mathcal{U} - \rho \left(\frac{1}{2} / \frac{dF}{d\zeta} \right)^2 + z \frac{dx}{dt}$$
(3)

and the surfaces on which the particles of the fluid move will be:

(1) wings:
$$f(x,y;t) = \alpha, \ g(x,y;t) = \beta$$
 (4)

(2) body:
$$Z = \int \chi(t)dt + t \cdot h(f,g) + q(f,g)$$
 (5)

where χ , c, h, q are arbitrary functions.

Theorem 2

To an analytic function of a complex variable $F(\zeta) = \varphi + i \psi_{\text{corresponds}}$ a class of three-dimensional stationary flow of an ideal incompressible homogeneous fluid:

$$v_x - i v_y = dF/d\zeta$$
, $v_z = \theta(\psi) (\theta: arbitrary function)$ (6)

Flow 6 is everywhere vortical:

$$\Omega_{\mathbf{x}} - \omega \Omega_{\mathbf{y}} = \frac{dF}{d\zeta} \cdot \frac{d\theta}{d\psi}, \quad \Omega_{\mathbf{x}} = 0. \tag{7}$$

Pressure:

$$p = c + \rho U - \frac{1}{2} \left| dF \right| d\zeta \right|^2 \tag{8}$$

the flow surfaces consist of two families:

(1) wing: $\psi(x,y) = a$.

(2) body:
$$Z = \theta(\dot{\psi}) \int \left\{ \left| d\zeta \right| dF \right|^2 dF \right\}_{\psi = a} + \omega(\psi).$$
 (9)

Theorem 3

In order that the surface z = V(x,y) + c can be considered as a fuselage for a given wing $\psi(x,y) = a$, it is necessary and sufficient that the function V be satisfied by the following equation:

$$D\left\{D(V,\psi)/D(x,y),\psi\right\}/D(x,y)=0. \tag{10}$$

Theorem 4

The set of all fuselages, represented parametrically:

$$x = \alpha + \frac{\partial W}{\partial \beta}, \ y = \beta - \frac{\partial W}{\partial \alpha}, \ z = \alpha - \frac{\partial W}{\partial \beta} \tag{11}$$

possesses the property that the function $W(\alpha,\beta)$ satisfies the following 4th-order partial differential equation:

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where
$$D\left(\sum_{\alpha},\sigma\right)/D\left(\alpha,\beta\right) = 0$$

$$D\left(\sum_{\alpha},\sigma\right)/D\left(\alpha,\beta\right) = 0$$

$$A = \frac{2}{\Delta} \left[\left(1 - \frac{\partial^{2}W}{\partial\alpha\partial\beta}\right) \frac{\partial}{\partial\alpha} \frac{1}{\Delta} \frac{\partial^{2}W}{\partial\alpha^{2}} - \frac{\partial^{2}W}{\partial\beta^{2}} \frac{\partial}{\partial\alpha} \frac{1}{\Delta} \left(1 + \frac{\partial^{2}W}{\partial\alpha\partial\beta}\right) \right] + \frac{\partial}{\partial\beta} \left[\left(\frac{1}{\Delta} \frac{\partial^{2}W}{\partial\alpha^{2}}\right)^{2} + \frac{1}{\Delta^{2}} \left(1 + \frac{\partial^{2}W}{\partial\alpha\partial\beta}\right)^{2} \right]$$

$$B = 1 + \frac{4}{\Delta^{2}} \left[\left(1 + \frac{\partial^{2}W}{\partial\alpha\partial\beta}\right)^{2} - \Delta \left(1 + \frac{\partial^{2}W}{\partial\alpha\partial\beta}\right) + \left(\frac{\partial^{2}W}{\partial\alpha^{2}}\right)^{2} \right]$$

$$\left(\text{where } \Delta = 1 + \frac{\partial^{2}W}{\partial\alpha^{2}} \cdot \frac{\partial^{2}W}{\partial\beta^{2}} - \left(\frac{\partial^{2}W}{\partial\alpha\partial\beta}\right)^{2} \right).$$

Examples

Consider the function $F = \frac{m}{2\pi} ln \zeta$, corresponding to a source in two-dimensional motion. Here the wing can be taken as the 2x-plane, and the fuselage as a paraboloid of rotation. The flow lines are defined by the intersection of the planes, passing through the 2x-axis, with the paraboloid. The projection of flow on the 2x-plane is the well-known potential two-dimensional flow near the source. The original article, available in CIA, shows a photograph of a model of the flow surfaces for this function.

Consider another simple example: $F = \frac{\Gamma}{2\pi i} \ln \zeta$. Here the wings are circular cylinders (with the z-axis), and the fuselage is a helicoid. The projection of flow on the xy-plane is the well-known planar flow near a point source: The original gives a photograph of the model.

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